A vibrating system is a system that can store energy in at least two forms: in the case of mechanical oscillators the energies are kinetic and potential. The oscillation involves a periodic transformation of energy between them. Since the efficiency of any energy transformation is always less than 100%, at each cycle some energy is dissipated and free vibration decays in time, unless the system is supplied of some energy to sustain vibration.

All real vibrating systems thus dissipate energy in some way. The simplest element that is used to model the energy dissipation or damping is the **viscous dashpot**.

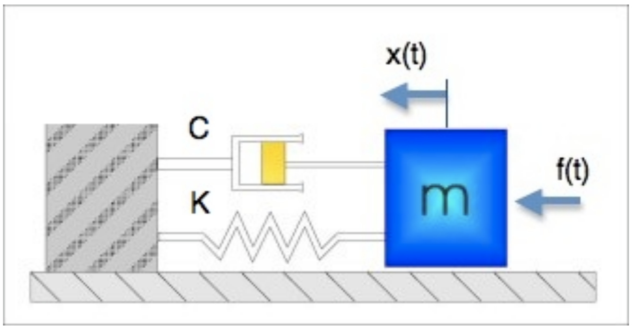

Fig. 1: Single degree of freedom system, Mass-Spring-Viscous dahpot

Its force is linearly proportional to velocity and therefore the equation of motion of a single degree of freedom (SDOF) system, as depicted in **Errore. L'origine riferimento non è stata trovata.**, can be written as

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{1}$$

where $x(t)$ is the generalized coordinate defining the configuration of the system, $f(t)$ is an external input acting on it and m, k, c are three constants defining inertial, elastic and damping properties. Close inspection of the behaviour of real structures suggests that the viscous damping model is not very representative when applied to multi degree of freedom systems. In fact, dynamic tests on different materials have shown that the energy dissipated for every cycle of oscillation is independent of the frequency of oscillation.

An alternative theoretical damping model is provided by the **hysteretic** or **structural** damper, which not only has the advantage that the energy lost per cycle is independent of frequency, but also provides a much simpler analysis for MDOF systems. The equation for the hysteretic damping model is obtained introducing

$$c = \frac{d}{\omega} \tag{2}$$

where d is the hysteretic damping coefficient, ω the frequency of vibration. Substituting

$$m\ddot{x} + \frac{d}{\omega}\dot{x} + kx = f(t) \tag{3}$$

The drawback of this model is that its solution of free vibration is a-causal, that is it predicts a response before an event is applied: demonstration of this, suggested by Crandall in his dissertation 'The role of damping in vibration theory', is discussed in detail in the Chapter 2 of this work.

There is thus scope for investigating novel models that satisfy both the frequency-independent and the causality requirements. A model that could solve the problem consists of a spring in parallel with a large number of spring-damper elements in series. The model is shown in Fig. 2 and it is sometimes referred to as *Maxwell-Weichert model.*

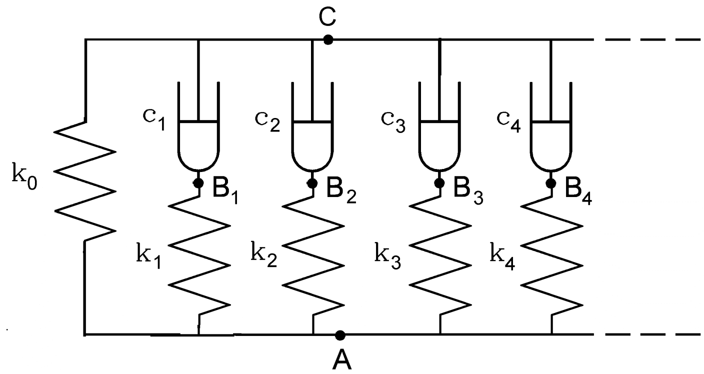

Fig. 2: Maxwell-Weichert model

Study on this model with only one spring-damper branch in parallel with a spring (called *Zener model*) is already been carried out by Carrella in his dissertation 'On the Dynamic Behaviour of a Mass Supported by a Parallel Combination of a Spring and an Elastically Connected Damper', where he, with the other three authors, Brennan, Waters and Vicente Lopes, presented a consistent and concise analysis of the free and forced vibration of a mass supported by a Zener model.

The generalization of the problem to a mass supported by a Maxwell-Weichert model, instead, was carried out by Genta in his dissertation 'On the Equivalent Viscous Damping for System with Hysteresis' where he provided an analytical way to extract the modal parameter from a generic multi degree of freedom system.

Deepen the analytical aspects of the Maxwell-Weichert model and to realize numerical script to apply the model to a generic system, will be the main objects of this dissertation.

In the Chapter 2 we will go over the modal model and the response model of systems with viscous and hysteretic damping. We will analyse how we can find the modal parameters and the receptance equation for system with one or more degree of freedom. We will analyse advantages and disadvantages of each damping model, and we will introduce an alternative-damping model which give us the possibility to describe the real behaviour of the structures with a correct and simple mathematic formulation.

In Chapter 3 we will show the practical differences comparing the Frequency Response Function and Transmissibility Function plots of both viscous and hysteretic damping models. Then, modal parameter extraction methods, used during experimental tests, will be analysed. In the end, focusing on the viscous damping model, we will analyse an iterative method to find the value of the damping factor, c, to obtain a damping ratio, ζ, constant with frequency, and we will show the duration of the computational algorithm.

After a detailed discussion about the classic damping model, in the Chapter 4 we will first go over the Zener model applied to a SDOF system, then to a MDOF system: we will analyse the method that give us the possibility to extract the modal parameters from a general system with relaxation damping, and we will show the constancy of the damping ratio, ζ, with the frequency. We will then focus on the Frequency Response and Transmissibility Functions of the system, analysing the differences that exist between these and the classic models ones.

In the chapter 5, the work is going to conclude with a discussion of the results obtained during the work and suggestions for future developments.

The work, mainly analytical, has been developed with the help of Matlab, and in the Appendix A we can find some Matlab script realized by the author.